Assessing the sound of a woodwind instrument that cannot be played

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ABSTRACT

Historical woodwind instruments in museums or private collections often cannot be played, by virtue of their poor condition or the risk of damage. Acoustic impedance measurements may usually be performed on instruments in good condition without risk of damage, but only if they are in playable condition: complete, with functioning mechanism, well-sealing pads and no open cracks. Many museum specimens are not in this condition. However, their geometry may almost always be accurately measured, and the measurements used to calculate the acoustic impedance as a function of frequency via a computer model of the body of the instrument. Conclusions may then be drawn about the instrument's pitch, intonation, temperament, fingerings, effects of bore shrinkage and even the timbre of the notes. A simple linear, plane- and spherical-wave computational model, originally developed for calculating the acoustic impedance of conical-bore woodwinds, is here applied to bass clarinets for the first time. The results are assessed by experimental impedance measurements and by playing tests on an historical Heckel bass clarinet in A of 1910 that has been continuously maintained in playing condition but has been relatively lightly used. The degree of agreement between the acoustic measurements and the calculations, the required measurement accuracy and the potential and limitations of the method are discussed, and specific conclusions for this instrument are drawn. Measurement of the frequencies produced in playing tests allowed us quantitatively to estimate the effects of mouthpiece and reed on the pitch of the produced notes. The method is shown to be a viable method for the examination of historical woodwind instruments.

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1. Introduction

The aim of the investigations in this paper is to test the idea that it is possible to model the input impedance of a woodwind instrument sufficiently accurately that one may draw reliable conclusions about its behaviour purely from geometrical measurements of its bore, tone holes and keypads. This will enable the vast collections of woodwind instruments in museums to be used for primary evidence of their sounds without risk of damage.

There is a very large number of musical instruments in museum collections in the UK alone. These are steadily being catalogued in the MiniM database, which contains 20,000 records so far [1]. Clearly, a very important property of a musical instrument is its sound and related questions such as its pitch, temperament and fingering. However, the overall responsibility of museums is to protect and promote the tangible and intangible natural and cultural heritage [2], and many institutions preclude playing the instruments because of the risk of damage [3]. This is especially

true for woodwind instruments where the act of playing rapidly introduces air at a much higher humidity and temperature, triggering potentially damaging reactions in the wood. Moreover, even if playing is permitted, it is fairly unlikely that a wind instrument 150–200 years old will be usefully playable without restoration that goes well beyond normal conservation. Sealing against leaks is crucial in these instruments; 200-year old pads – when original – are likely to leak, and cracks in the wooden body are quite frequent. These leaks strongly affect the acoustic impedance, rendering the instrument useless for the assessment of its musical potential either by playing or by acoustic impedance measurement. Occasionally a museum will permit full restoration for playing on a special occasion. Examples are the Brussels Musée des Instruments de Musique, where an Adolphe Sax instrument (B.B. mim.2601) was partially overhauled to play in the Sax bicentenary celebrations in 2014, and the Robert Schumann School in Düsseldorf, where a Stengel bass clarinet in A, originally owned by the Bayreuth Theatre and used in some original Wagner operatic performances, was restored for a demonstration concert and future use [4]. But the great majority of wind instruments in museums

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remain musically hidden from players or from direct sampling of the sound.

However, museums will normally permit the handling and careful measurement of instruments that are not too fragile, by an accredited researcher under supervision and the guidelines of ICOM/CIMCIM [5]. This has been used to study the development of types of musical instrument and their keywork (see, for example, [6–8] for clarinets), but their sounds have so far been mostly inaccessible.

The principle upon which the main methodology of this paper is based is that the sound of a wind instrument is dominated by the shape of its air column, as indicated by its input impedance. Although there are (and probably always have been) endless arguments about the influence of materials on a wind instrument, it has been demonstrated that the energy radiated by the walls of the tube into the room is inaudible in comparison with that emitted by vibration of the air column, and that wall material has little audible effect, as long as it is reasonably dense and has low porosity [9–11]. There is also very clearly an acoustic cooperation between the mouthpiece/reed and the resonator or air column, and the mouthpiece is of great importance for the details of the timbre and for the ease of playing. However, the importance of the resonator is shown by the observation that the character of the instrument appears mostly to go with this rather than with the mouthpiece [12]: a clarinet-type mouthpiece of suitably small volume works reasonably well on an oboe; the instrument still sounds like an oboe not like a clarinet, and it overblows an octave not a twelfth [13,14]. In this paper we are concentrating on the resonator. Its acoustic properties are defined by the sets of resonance, or impedance, peaks that it possesses and the relationships between them. If we can understand the influence of the detailed shape of the air column on the sound production, for all notes and all relevant frequencies, we shall know a great deal about the nature of the instrument. Furthermore, this knowledge is objective, and not subject to the physiology or prejudices of any player.

A well-preserved instrument from 1910 was used to make quantitative comparisons for this trial. Standard acoustic computational methods (described below) were used to calculate the impedance spectrum for each note of the instrument, and two tests of the accuracy were performed: one by measuring the input impedance directly in the laboratory, and the other by playing tests on the instrument, measuring the frequency of the note emitted at each fingering and looking at the predicted intonations produced by both 'normal' and 'alternative' fingerings. Thus we investigate two questions: can we calculate impedance spectra with sufficient accuracy without playing the instrument, and does this give significant musical information about the instrument?

2. Modelling of woodwind instruments

The development of mathematical and computational methods of modelling woodwind instruments has taken place over more than a century, beginning with the analytical ideas of Hemholtz [15]. Major contributions were made by Bouasse [16] and by Benade and his collaborators [10]. The understanding of woodwind acoustics progressed through analytical expressions for lossless and then lossy systems [17–19], linear system calculations [20], analysis of the reed/mouthpiece system [e.g. [15,21–23], impedance of the bell [24,25], non-linear treatment of the reed generator [26] and other factors; an excellent recent treatment appears in Chaigne and Kergomard [27]. In 1979, Plitnik and Strong [28] first applied the computer modelling method to the whole instrument. They split the bore (of an oboe in this case) into short cylindrical segments, thus approximating the conical shape of the bore by

the staircase approximation, started from the calculated impedance of the bell radiating into open air and summed each complex impedance, in series for the segments and in parallel for the tone holes. A reed cavity impedance was added in parallel at the end of the sum. The result was the spectrum of impedance peaks as a function of frequency over the audible band. Note that this and most other approaches are based on linear theory and strictly only apply to small amplitudes. The non-linear effects of large amplitudes are critical in the understanding of the peaks selected, as discussed below, but there is agreement amongst all authors cited that linear acoustics suffices for the calculation of the tube resonances.

This general approach is still used today. Developments since Plitnik and Strong include improvements to the expressions for tone hole impedances, for wall losses, for the radiation impedance of the bell, for the influence of the reed generator and in the matrix formulation (analogous to electrical transmission line theory) which significantly speeds up the calculation [29,34,35]. Nederveen [30] has added valuable insight into the elements of the modelling equations and a number of experimental measurements. Research on simulating clarinet and saxophone sounds dynamically using digital formulations of the air column and reed/mouthpiece system in the time domain are also reaching an interesting stage [23,31,32,34].

Two computer implementations of linear acoustic modelling have been made more widely available and are cited in the literature. The program IMPEDPS was written by Robert Cronin in the 1990s, based on the developments and equations given by Keefe [29] and by discussions with Keefe and Benade. RESONANS was developed around the same time by IRCAM and the acoustics department of the Université du Maine in Le Mans (a brief note on application to recorders is given by Bolton [33]). Valuable summaries of the necessary equations for each component of the transmission line matrix formulation have been given by Scavone [34] and more recently Yong [35].

It turns out that the methodology descended from Plitnik and Strong is quite general for woodwind instruments that have reed generator excitation. It may also be used for flutes and recorders by using admittance peaks rather than impedance peaks, since the open entry ends of air-driven oscillators require a pressure node rather than antinode at the entry end. We have therefore used the methodology to test the basic assertion, that acoustic impedance spectra can be calculated by geometric measurements on instruments to sufficient accuracy to give musically useful information. We first review the advances in understanding of woodwind instruments that have been made by both experimental and theoretical modelling of impedance spectra.

2.1. Applications of impedance spectra to the understanding of woodwind instruments

The understanding of the influence of impedance spectra came first through experimental measurements and approximate analytical solutions of the acoustic equations, with particularly notable contributions made by Benade (summarised in [10]), Backus [20,54] and their co-workers. Indeed, the increased understanding of instrument acoustics provided by measurements and calculations of input impedance led Benade directly to a new design of clarinet bore and keyhole placement, in which inaccuracies in intonation were corrected by enlargement or contraction of the bore around pressure nodes [36–39]. Clarinets to the 'Benade NX design' are manufactured by Stephen Fox Clarinets (Toronto) [40].

There are various different ways in which input impedance can be measured experimentally. Dalmont [41,42] provides a comprehensive review of input impedance measurement techniques developed during the 20th century. One of the most popular

approaches (pioneered by Benade and Backus) exploits the direct definition of impedance as the ratio between input pressure and acoustic volume flow. The approach involves passing an acoustic signal generated by a loudspeaker through a high impedance capillary and then into the instrument under investigation. The volume flow entering the instrument can be determined from the pressure in the cavity between the loudspeaker and the capillary. By then measuring the pressure at the entrance to the instrument, the input impedance can be deduced. In early capillary-based measurement systems, a feedback loop was employed to maintain the cavity pressure (and therefore the volume flow injected into the instrument) constant, such that the pressure measured at the entrance to the instrument was directly proportional to the input impedance. In more modern systems, the cavity pressure is determined as a function of frequency, either using a second microphone positioned within the cavity, or via calibration. A very compact and portable capillary-based system has been developed by the Institute of Musical Acoustics, Vienna; this system is commercially-available and is known as BIAS (Brass Instrument Analysis System). As its acronym implies, it was originally developed for brasswind, and later modified for woodwind [43] and it has been applied in the quality control of brasswind instruments since 1989 [44]. The knowledge and application of impedance and other scientific measurements to instrument manufacturing has been assisted in recent years by the Pafi collaboration (Plateforme modulaire d'aide à la facture Instrumentale, [45]) in France, which seeks to make scientific measurements, including input impedance, available to small manufacturers together with tools to predict the effects of changes.

Another method of measuring acoustic impedance is acoustic pulse reflectometry. In this method, an acoustic pulse is sent into the instrument under test and the reflected signal is measured. Analysis of the reflected signal enables the input impulse response of the instrument to be determined, from which both its bore profile and input impedance can be calculated. Details of this method can be found in [46–48].

Many of the studies have been made primarily to test the modelling theory, rather than to investigate modern or historical instruments themselves. Campbell has written a review of the acoustic evaluation of wind instruments but the entry on woodwind instruments is very short [49]. The only acoustical investigations of historic clarinets appears to be the work of Jeltsch and co-workers. Jeltsch, Gibiat and Forest were able to perform acoustic impedance measurements on a set of four six-key clarinets made by Joseph Baumann (fl. Paris, c. 1790 – c. 1830) [50]. The set was in very good condition and playing was permitted, so they could compare impedance measurements with playing frequencies, and also make comparisons with a modern (Noblet) clarinet. The set of historical clarinets was particularly interesting, since their maker supplied the distinguished clarinettist and pedagogue Jean-Xavier Lefévre, who refers to these clarinets in his famous tutor [51] and gives particular fingerings to exploit or overcome their characteristics. In their data analysis they concentrated on the harmonicity relations produced by the fingerings of the clarinets. They showed, for example, that the first register was not well tuned, and also invented the concept of 'impedance maps', which clearly show the tuning and harmonicity relationships in the Baumann instruments. Lefévre remarked on the tuning in his tutor and also composed his sonatas mainly in the second register of the instrument. The modern clarinet showed much better alignment of the harmonics. We use and develop the impedance map concept further in the present study (Section 5.3). Jeltsch et al. also observed that higher notes of the instruments were supported by apparently random combinations of resonances; we shall also return to this point in Section 5.3. Jeltsch and Shackleton have

performed a similar study on early nineteenth century clarinets by Alexis Bernard and Jacques Francois Simiot [52].

The first application of computational impedance modelling to a complete instrument was by Plitnik and Strong in 1979 [28] to the oboe. Their main concern in the modelling was to demonstrate the close agreement between calculated and measured impedances. This indeed was found, with peaks being accurately located and peak shapes in good agreement, though the peak-to-valley ratios in the experimental measurement were typically a factor of 2 lower than in the simulation. They ascribed this to unaccounted-for losses, in particular pad and finger resilience, socket junctions and sharp corners of tone holes, and we should expect similar discrepancies in the case of clarinets. They investigated a single oboe and were able to demonstrate why certain notes were 'bad' and why certain alternative fingerings worked. No application to historical instruments was made. Soon after, Schumacher [53] developed the theory of the clarinet to include the reed/mouthpiece generator and used a similar computational approach to Plitnik and Strong. He tested the theory on the experimental measurements of Backus on a single clarinet [54] and obtained similarly good agreement.

In the 1990s the IMPEDPS program was written by Cronin and applied to the understanding of the behaviour of fingerings and auxiliary fingerings on modern and replica baroque bassoons [55,56]. He was able to demonstrate the reasons for 'surprising' fingerings shown in contemporary fingering charts for the baroque bassoon, hence was able to obtain useful information on historical instruments by impedance calculations.

One of the authors of the current paper, Dart, himself a maker of reproduction baroque bassoons, applied computational impedance modelling to the study of historical instruments in museums, in 2011, also using IMPEDPS [57]. He examined approximately 80% of surviving baroque bassoons, making detailed internal measurements of thirty-six and computing impedance spectra. This enabled him to compare stylistic traits, to establish a new typology of baroque bassoons and to study eighteenth-century woodwind construction processes and tooling. He was also able to discover connections between an instrument's internal design and its probable playing characteristics. In two cases of incomplete historical instruments, he reconstructed the design and then built replicas of each. He found them to have different playing characteristics which could be understood in terms of their calculated acoustic impedance spectra.

In an investigation reported in 2012, Hichwa and Rachor [58] used similar acoustic models to Keefe in a new program designed to investigate the effects of geometry in more detail, and to apply mathematical analysis to the results. From measurements of 44 original bassoons and 14 reproductions from the baroque and early classical period, they were able to deduce the temperaments used by the original makers, which clustered in identifiable classes around mean-tone temperament. They showed by analysis how best the boot joint can be made to aid intonation. They were also able to identify acoustic inadequacies in some of the original designs, normally in the wing-joint, thus aiding the period-instrument maker in the selection of instruments to reproduce.

Dalmont, Gazengel, Gilbert and Kergomard have assessed clarinets, alto saxophones and oboes [59], using both impedance measurements and the RESONANS software, and reached valuable conclusions about the quantitative influence of the reed impedance, the placement of the register hole, and the measurement and effect of inharmonicity in the resonances.

Sharp and co-workers have applied impedance measurement by both the capillary system and acoustic pulse reflectometry (see Section 4.2) to the question of consistency of large-scale manufacture of woodwind instruments: trumpets [60], oboes [61] and

clarinets [62]. In the case of oboes, for example, significant playing differences between instruments were found to be caused by relatively minor variations, such as in the venting height of one key, indicating that instrument variability can be at least partly due to the final regulation of the instrument. However, there were also larger quality-control differences such as variations in the bore profile.

3. Computational methodology

Our approach has been based largely on the equations developed by Keefe [29], and uses his expressions for the impedance of conical segments including thermal and viscous losses, and for tone holes (closed, open and open with a key pad at a certain distance above the hole). Keefe's paper includes most of the advances made in theoretical modelling since Plitnik and Strong and was verified by experiments made by himself and Cronin. It is a linear, small signal plane- and spherical-wave approach. We shall discuss and cite sources for the key parameters and the necessary equations. These are, the input constants, the radiation impedance of a bell, the impedance of a conic section (with thermal and viscous losses at a smooth wall), the tone hole impedances (open, closed and with a pad above) and the reed impedance.

3.1. Input parameters

The following parameters were used:

Speed of sound:	347 m s^{-1}
Density of air	1.19 kg.m^{-3}
Viscosity of air	$1.85 \cdot 10^{-05} \text{ Pa.s}$
Specific heat ratio C_p/C_v	1.4
Thermal conductivity of air	$2.63 \cdot 10^{-02} \text{ W.m}^{-1}.\text{K}^{-1}$
Specific heat at constant pressure C_p	$1.006 \text{ J.kg}^{-1}.\text{K}^{-1}$

The parameters above were chosen for appropriate playing conditions, that is, a somewhat elevated temperature (27 °C) and humidity and a substantially elevated CO_2 content of the exhaled air [30]. The laboratory measurements were made under normal laboratory conditions, approximately 20 °C and normal atmospheric composition. Coincidentally but conveniently, the product of air density and speed of sound (which determines resonant frequencies) for these two conditions agree to better than 1 part in 8000. This is approximately 0.2 cents, below the limits of audible perception, so we need no corrections when comparing theoretical and experimental data.

3.2. Radiation impedance of a bell

The precise calculation of the radiation impedance for a duct termination of various shape and flare is the subject of many papers (for example, [24,25,27,63]). As noted by Chaigne and Kergomard [27], p. 684], there are no straightforward formulas for the radiation impedance of a cone or flared bell; however, in a detailed spherical-wave treatment, Hélie and Rodet [63] have given an analytic (but computationally intensive) expression for the radiation impedance of a segment of a pulsating sphere, which should model a bell quite accurately. Dalmont, Nederveen and Joly [25] have experimentally investigated short, rapidly-flaring catenoidal bells and their approach may be applicable to at least some clarinets. Importantly, their results show that the overall input impedance of a clarinet-like tube is only weakly influenced by the radiation impedance of the bell. This might be expected since one purpose of the design of the bell is to reduce its radiation impedance; moreover, the values of the radiation impedance of the bell are some

three orders of magnitude lower than those of the overall instrument impedances, and in any case have little influence after the bottom notes in each register. We have investigated this question in the 'bell note' cases by calculating the impedance spectra using (a) the semi-empirical formula due to Levine and Schwinger [24], (b) the expression due to Hélie and Rodet (Eq. 23) and (c) the empirical formula due to Benade and Murday [64]. The only difference was a less than 5% change in the amplitude of some of the impedance peaks, with no detectable change in their frequency, in the 20 – 2000 Hz range of our calculations. We have therefore chosen to use the empirical formula of Benade and Murday, which has the benefit of experimental derivation and very efficient computation. Both tone holes in cylindrical bodies and radiating tubes with finite flanges are covered, and they give empirical formulas for the end correction. This is converted into impedance by the standard formula for a lossless cylinder (e.g. [65]), since there are no walls to cause losses.

3.3. The impedance of a conical segment

Eq. (21) of Keefe's 1990 paper [29] on the modelling of woodwind air columns was used. This is a spherical wave solution, and includes viscous and thermal losses at a smooth wall. The wall losses are averaged by putting them equal to the loss at the centre of the conical segment, but since the losses vary with radius it is then essential to keep the segments short. A difference of less than 10% between the end diameters of the segments was used. Kulik has proposed an analytic solution to the 'long cone with losses' problem [66] that offers much faster computation. However, this has been criticised on physical arguments by Grothe [67], who also finds that it does not converge to the staircase or multi-conic models. Grothe does, however, show that an improved solution could be possible using the work of Nederveen [30].

3.4. Tone hole impedances

Eq. (3) of Keefe's 1990 paper [29] on the modelling of woodwind air columns was used, with effective length corrections as given in his Eqs. (5)–(9). These depend on both theory and on experiments by Benade and Murday [64] and by Cronin and Keefe [unpublished]. These give the series and shunt impedances of open and closed toneholes and include viscous and thermal losses and the presence of a pad above the hole. Following Cronin we divide the series impedance of the tone hole equally between the tone hole itself and the bore segment. The "flange" of the open hole is taken as the cylindrical body of the tube and a correction is included for the corner radius of the outside (but not the inside) edge of the hole.

Several authors have published theories and/or experiments on tone-hole impedance since 1990: Nederveen et al. [68], Dubos et al. [69] and Dalmont et al. [70]. However, all these authors state that the accuracy of the experimental measurements is at present insufficient to distinguish between the theoretical models. Dalmont et al. and also Yong [35] provide figures for the length corrections on the different theories showing that the differences are not large. Moreover, the above papers mainly treat open or closed tone holes. The only information for tone holes covered with a key or plateau that appears to have been published since Keefe's paper of 1990 is that of Dalmont, Nederveen and Joly [25]. However, they do not include the case of a tone hole in the side of a cylinder; such holes comprise 22 out of the 24 holes on this bass clarinet. We have therefore retained Keefe's expressions and the experimental data of Benade and Murday [64], used originally in the IMPEDPS program, in our work.

We do not include external interactions between tone holes in this model, in common with Plitnik and Strong [28], and Cronin

[55]. This may be a source of error, but interaction equations do not appear to be available for key-covered open holes. We expect the effect to be relatively small for bass clarinets, with widely-separated and covered holes. The work of Lefebvre et al. [71] suggests that the error will be to flatten the computed resonant frequencies by perhaps a few cents.

3.5. Reed impedance

The reed volume (including an estimate of the average vibrating part of the volume) should in principle be accounted for as a complex impedance in parallel with that of the column, since the oscillation forces the reed away from the mouthpiece lay [30,72]. More accurately, we should need the impedance as seen from the mouthpiece looking at the reed, whose imaginary part should be equal and opposite to that of the resonance peak (to ensure that there is no phase shift around the feedback loop to the reed). This therefore includes a contribution from the mouth and oral cavities. Hence, the frequencies selected by the instrument will be slightly below the impedance peaks of the tube alone, even when including a segment of equivalent volume to the mouthpiece.

Benade and Gans [73] showed that the shift is calculated by balancing the phase shift between pressure and flow in the mouthpiece with that arising from the inertia and stiffness of the reed and that of the oral cavity. This has been considered by Nederveen [30] and by Dalmont and co-workers [59]. The latter have reported theoretical and experimental work on soprano clarinets, oboes and alto saxophones using an artificial mouth with a blowing machine. They show that reed impedance effects can be satisfactorily incorporated in an impedance model by adding a frequency-independent equivalent length correction to the end of the tube (including the mouthpiece volume). For soprano clarinets, this end correction was found experimentally to be 7 ± 2 mm, somewhat smaller than Nederveen's estimate of 10 mm for the length correction itself, plus a further 5 mm for a correction due to reed damping. No estimates have been reported on bass clarinets to our knowledge, but an expression is given by Chaigne and Kergomard [[27] Eq. 9.17] from which we may estimate the scaling factor. They give the 'embouchure equivalent length', Δl as

$$\Delta l = \frac{\rho c^2}{p_M} \frac{S_r}{S} . H \qquad (1)$$

where ρ is the density of air, c the speed of sound, p_M the mouth (closure) pressure, S_r the reed area, S the bore area and H the slit opening of the reed when not under pressure. In comparison to a soprano clarinet, a bass clarinet of the same pitch class scales linearly in its length and linearly in its bore area (not diameter). Its mouthpiece thus has typically twice the reed area, twice the bore area, twice the slit opening and a similar mouthpiece pressure (resulting in a greater air flow through the larger aperture). The value of Δl can then be roughly estimated as around double the correction in soprano clarinets, namely 14 ± 4 mm, which should be increased by about 6% (0.84 mm) in the present case since it is an instrument in A. This estimate is not accurate enough to incorporate immediately in the computations (in fact, Dalmont et al. suggest using this length as a fitting parameter) but will be discussed after presentation of the results. In our implementation we use an equivalent length correction as suggested by Dalmont et al.

3.6. Verification and performance of the program

The IMPEDPS program and its source code was made available to us. We were able to configure our program, written in MatLab ™, to have identical implementations of the parameters and equations, and thereby verify that the outputs were indistinguishable

within computational precision. This gives the ability to calculate a complete instrument (50 notes including alternatives) and to analyse its resonances in about one minute [1], and also gives the facility to introduce different acoustic models. For example, it was straightforward to introduce and calibrate an embouchure equivalent length, following Dalmont et al. [59] and this was adopted.

3.7. Selection of harmonics by the instrument

Once an impedance spectrum has been calculated the next stage is to discover which peaks are actually excited. This uses a most important principle: the possible resonant frequencies in a reed-driven air column are those for which the input impedance of the column is a maximum. This principle was established by Benade (1966) building on work of Bouasse [16] and it contradicted the long-held theory, summarised by [17], that the mouthpiece/reed system generates a broad sound spectrum from which the air column resonances filter out the tone that is heard. Benade showed in some cleverly-designed experiments that this was not how a reed instrument worked. This and other principles are discussed in Benade and Gans [73] and in expanded mathematical detail in Worman [26]. An essential feature of the interpretation is that the reed generator is intrinsically non-linear and therefore necessarily generates harmonics, as does any non-linear oscillator. The amplitude of the harmonics above the fundamental necessarily increases as the blowing pressure increases (until the reed starts to close on the mouthpiece). Hence, if there are several impedance peaks in the tube spectrum *that are harmonically related*, these will all cooperate in generating standing waves when they are reflected back to the reed. This 'mode locking' effect will stabilise the oscillation, and was termed by Benade a 'regime of oscillation' following the earlier work of Bouasse. As the amplitude increases, the influence of upper harmonics also increases (initially by the nth power of the amplitude of the fundamental). This both further stabilises the note and adds to the richness of the harmonic spectrum. One consequence is that if the impedances are slightly stretched from true harmonic relationship then the note will sharpen as the amplitude increases, and flatten if they are compressed.

The harmonic spectrum and its stability thus has a complicated dependence on blowing pressure as well as on the basic clarinet resonances at a particular fingering [36]. For the purposes of this paper, we simply look for a good match between the first and at least one other resonance with harmonics of the pitch of the note being played, in the first register; on the clarinet these will be the third and if possible the fifth harmonics. In the second register, it is the second resonance peak that aligns with the fundamental of the sounded frequency, since the register key shifts the first peak out of 'alignment' with the harmonics so that it can no longer participate in a regime of oscillation. In the third register, the third resonance peak takes over this function. The cutoff phenomenon in instruments with tone holes and a bell (see [10,30,36]), whereby frequencies above cutoff do not reflect at the finger holes or the bell but pass through into open air, means that higher frequencies are unimportant in maintaining oscillation, though they can weakly affect the tonal colouration. We note that this effect is roughly twice as significant in clarinets than in bassoons, oboes or saxophones because of the absence of even harmonics, especially at low pitches, in the clarinets. As a rough rule, notes above written G in the second register (i.e. notes above sounding pitch E4, approximately 330 Hz) have all their harmonics above the nominal cutoff frequency. We shall examine the cutoff phenomenon in more detail in Section 5.3.

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[1] On a MacBook Pro with 3 GHz Intel Core i7.

4. Materials and methods

4.1. Description of the instrument and measurements

The instrument used for the tests was a Heckel bass clarinet in A from 1910 shown in Fig. 1 owned by one of the authors (DKB). It is a 21-key system including 5 plateau keys (holes I and IV are open fingerholes), and is German or Albert system with a so-called patent C#. In total, 22 of the 24 holes are covered by keys or plateaux.

Dated at 1910 from Heckel records [74] and formerly owned by the Kiev Symphony Orchestra, this has been kept in playing condition all its life, but only lightly played, no doubt as a consequence of there being relatively few orchestral parts for the bass clarinet in A [75]. It has been recently repadded with leather pads similar to the originals and is in very good playing condition. It has a straight bell, so there is no need to consider complications due to a curved bell. The effect of the curve of the crook may be estimated from data given by Félix, Dalmont and Nederveen (2012) [76]. The minimum radius of curvature (tube internal radius/bend radius) in this particular crook is $\kappa = 0.38$, and from their Fig. 4, the length correction will be at maximum approximately 0.8 mm. We have neglected this quantity in the calculations at present, though it is automatically taken into account in the empirical embouchure correction discussed below.

Bore diameters were measured with a large set of graduated circular Tufnol discs on the end of aluminium tubes. There was no sign of ellipticity due to shrinkage. The bore is 23.2 mm for all its length, with a largely-conical flare beginning 153 mm from the bell. It is therefore a good experimental instrument for this project. The mouthpiece is not original, but made by E. Pillinger closely to the dimensions of an original Heckel Bb bass clarinet mouthpiece in Nuremberg (D.N.gnm.MIR480, which have been published by Bär [77].

Tone hole positions were measured with an EC Class II tape measure, further checked against a calibrated 600 mm vernier height gauge, to 0.5 mm always referenced from the end of a joint; tone hole diameters and depths and bore disc diameters were measured with a SPI 30-440-2 (Super Polymid-Fiberglass Reinforced Plastic) caliper with accuracy ± 0.1 mm. In addition to the tone hole centres and diameters (measured both along and across the clarinet axis), the chimney depth, diameter of the body at the tone hole position, the diameter of the tone hole keypad (where fitted) and its opening height were measured. The radius of curvature of the outer tone hole edges was estimated at 1.0 mm. These parameters enter into the expression for the tone hole impedance when opened. Approximately 300 measurements in all were used to describe the instrument. We estimate that the parameters most affecting the tuning (the tone hole positions) are measured to approximately 0.3%, corresponding to an average tuning accuracy of ~5 cents. Since each length measurement is independent, this error applies separately to each note, and is not cumulative.

The mouthpiece and crook were measured by filling with water and weighing the water, taking the average of ten measurements, giving an estimated accuracy of ±0.5%. We do not know how closely the copy of this Bb mouthpiece is to the original supplied with the A clarinet. However, the results should be consistent between calculation and playing.

4.2. Experimental impedance measurement systems

Two systems were used to measure impedances in the laboratory: an Open University in-house single-microphone capillary system that has been extensively calibrated [78], and the commercial BIAS system [79]. A single measurement (G3) was made with the in-house system, which verified that the agreement between the methods was good. For all subsequent measurements the BIAS system was used. Both the BIAS and single-microphone measurement systems are capillary-based. That is, a capillary channel connects a controlled sound source to the entrance of the wind instrument to be measured. The capillary is designed to have an impedance that is frequency independent, and has a much larger magnitude than that of the air column being measured.

The general principle draws from determining two characteristic signals at each end of the capillary, which allows to obtain a good estimation of both the pressure and volume flow rate at the entrance of the measured instrument (one of which may be made constant using some active control). Provided the wavelength is sufficiently above the inner diameter of the instrument's bore, the ratio of pressure over flow rate gives the plane wave component of the impedance. Both systems are calibrated with a similar two-calibration method. The only difference between them is that the single-microphone calibration relies on the assumption that the cavity pressure remains the same regardless of the object being measured.

In contrast to a number of alternative, more accurate, impedance measurement systems, one advantage of capillary-based impedance measurement systems is that the apparatus can be made very compact. This is particularly useful in the context of the measurement of historical instruments, which often require the equipment to be transported to a museum. Furthermore, the measurement does not require post-processing and directly provides a sufficiently accurate impedance measurement over the frequency range of interest, which in our case is 20–2000 Hz. As shown below, the cutoff frequency beyond which standing waves are not formed in the instrument is approximately 1000 Hz in the Heckel instrument.

In the BIAS system [80–82] a chirp signal is sent to a loudspeaker while a microphone monitors the acoustic pressure in the cavity between the loudspeaker and the capillary. The envelope of the chirp signal is designed to compensate for the cavity resonances, such that the variation in the acoustic flow emerging from the capillary is minimised. By measuring the pressure amplitude recorded by a second microphone at the entrance to the air column under test, the input impedance magnitude can be determined. Impedance phase information can also be obtained from the system through the use of a phase meter connected to the two microphones.

In the Open University in-house single-microphone capillary system, there is no cavity microphone. Even though the cavity pressure is not monitored during a measurement, the apparatus is still able to provide accurate values of input impedance magnitude via prior calibration. Moreover, despite only incorporating one microphone, this set-up is also able to provide accurate measurements of input impedance phase [78]. However, unlike the BIAS system, the single-microphone system is an in-house design, whose set-up and operation is more cumbersome. This decreased ease of use can represent a considerable constraint for measurement of historical instruments at specific locations, which is why the BIAS system was preferred.

An adaptor was made from nylon to fit the BIAS system at one end and the crook socket of the bass clarinet at the other. The volume of the adaptor was made to be the same as that of the instrument mouthpiece at 28 cm^3, and the end fitted closely to the BIAS system. The instrument was therefore measured in the fully 'pushed in' condition, which refers to its sharpest possible tuning.

For any single measurement the appropriate note was fingered, while the BIAS system performed the frequency scan. It was evident during the experiments that the slightest inaccuracy in fingering or insufficient pressure on the pad, resulting in a tiny leak at the finger or pad, changed the amplitude of the impedances, espe-

Fig. 1. The Heckel bass clarinet in A used for the trials. (picture courtesy Huw Bowen).

cially that of the first resonance peak, quite drastically. Each measurement was therefore repeated after relaxing the fingering, to check that the two scans were essentially identical. This emphasizes the point made in Section 1, that the instrument must be in good, leak-free condition for meaningful impedance measurements.

4.3. Audio frequency measurements

In order to compare the measured and calculated impedances with the pitches actually produced, the instrument was played (after warming up), and the sounds recorded over chromatic scales. Each note was played for several seconds, without looking at a tuner and while attempting to play in the natural 'centre' of each note. Two sets of recordings were made, one with the mouthpiece pushed in (corresponding to the acoustic measurement conditions) and the other with the mouthpiece pulled out 10.8 mm, the maximum practical on this instrument, to attempt correction of the perceived sharpness when referred to A4 = 440 Hz. Recording was made in a 'dry' acoustic room (though not an anechoic chamber) with a Rode NT1A microphone (20 Hz–20 kHz), using an Akai EIE Pro interface and Logic Pro X software, at 24 bit 44.1 kHz. The resulting WAV files were segmented into sections for each note, each at least 4 s long after truncating the transients at the beginnings and ends of the note to leave a steady tone portion. The frequency was determined in MatLab ™ using the YIN algorithm [83]. The accuracy of this method is estimated by its authors to be approximately ±1 cent, which is much better than can be obtained by digital Fourier transform methods.

5. Results

5.1. Comparison of calculations and acoustic measurements

The tone-hole cutoff frequency for this instrument is about 1000 Hz, calculated from Benade's formula [10] for an open tone-hole lattice

$$f_c = 0.11c \left(\frac{b}{a}\right)\left(\frac{1}{sl}\right)^{1/2} \tag{2}$$

where f_c is the cutoff frequency, c the speed of sound, a the pipe radius, b the hole radius, s the hole spacing and l the acoustic length of the holes. Clearly this is an approximation, since the hole spacings and diameters do vary somewhat, but it is confirmed by visual inspection of the impedance spectra. It is worth noting this value, since for bass clarinets, and also by scaling from soprano clarinets, one would normally expect a cutoff around 750 Hz [10,73]. This could be a significant parameter to evaluate in the study of historical instruments, since it definitely affects the musical sound and playing qualities, as discussed by Benade [10], who notes that woodwind instruments have actually 'evolved' over the centuries so that their cutoff frequencies became approximately constant over the whole range of the instrument. Waves with frequencies beyond the cutoff limit are not reflected at the first open tone hole but transmit through to and out of the bell (which is designed to have a similar cutoff frequency). They thus do not contribute to

the standing waves in the instrument nor to the feedback that stabilises the oscillations of the reed, though they can contribute (weakly) to the sound spectrum. We thus chose the frequency range 20–2000 Hz, with 0.5 Hz steps, for both the measurements and calculations. The range on the instrument for analysis was chosen to be from written E2 to D5 (69.3–494 Hz fundamental frequencies), corresponding to C#2 to B4 concert pitches. Whilst information could be obtained from higher note fingerings. It is less significant; only one resonance frequency contributes to defining the pitch produced for notes above about G4, and this pitch can be varied widely by embouchure control in the altissimo regime. In this regime the pitch of the sound produced is more reliant on the skill of the player than on the instrument.

To give an overall impression we first show a sequence of notes from (written) C major arpeggios, from E2 up to C5, with experimental and calculated impedances superimposed (Fig. 2). No embouchure correction was made for these data. The experimental absolute values of the impedance peak amplitudes agree well in frequency with the calculated values but are up to a factor of 2 lower in amplitude. This is consistent with the results of Plitnik and Strong [28], and may indicate that some losses in the tube, such as fingers, pads, turbulence at edges, or wall porosity are not taken into account in the model. However there may be experimental reasons for the discrepancy, such as the smoothing algorithm used by BIAS, or the short measurement interval possibly being insufficient to excite high-Q resonances completely. We have not investigated this discrepancy further, since our primary interest is in the frequencies of the peaks.

The agreement between experiment and calculation can be tested in detail. Fig. 3a shows the departures from equal temperament for the calculated and measured impedance values and for the frequencies determined from the playing tests. To magnify and quantify the intonation variations we express them in cents, where the difference in cents between two frequencies f_1 and f_2 is $1200\log_2(f_2/f_1)$. This gives a deviation from a target pitch by an amount that is comparable over the whole range. As expected from the arguments above, the playing frequencies are slightly below the impedance peak values. It is also apparent that the instrument is playing somewhat sharp overall (relative to equal temperament at A4 = 440 Hz) and becomes sharper at higher notes. On discovering this, we repeated the calculations and playing tests for the instrument pulled out 10.8 mm at the mouthpiece (see Fig. 3b). As expected, this gives a useful correction to the intonation, and playing experience indicates that this is just acceptable for playing at A4 = 440 Hz, given the variation that is available by embouchure control especially in the upper notes.

There is scatter in Fig. 3a, but we see that the calculated peaks are close to the measured peaks but systematically a little higher in frequency. We also see that the playing frequencies are lower still (as expected from acoustic theory) but appear to follow the measured or calculated deviations. Again, these can be further quantified. Fig. 4a shows the differences between calculated and measured impedance peaks, with the calculated peaks being a little higher in frequency. The difference averages at 10 ± 8 cents (±3 × the standard deviation of the mean), which can be corrected quite well with a 3 mm calibration correction added to the mouthpiece length (see below). It is possible that at least some of this
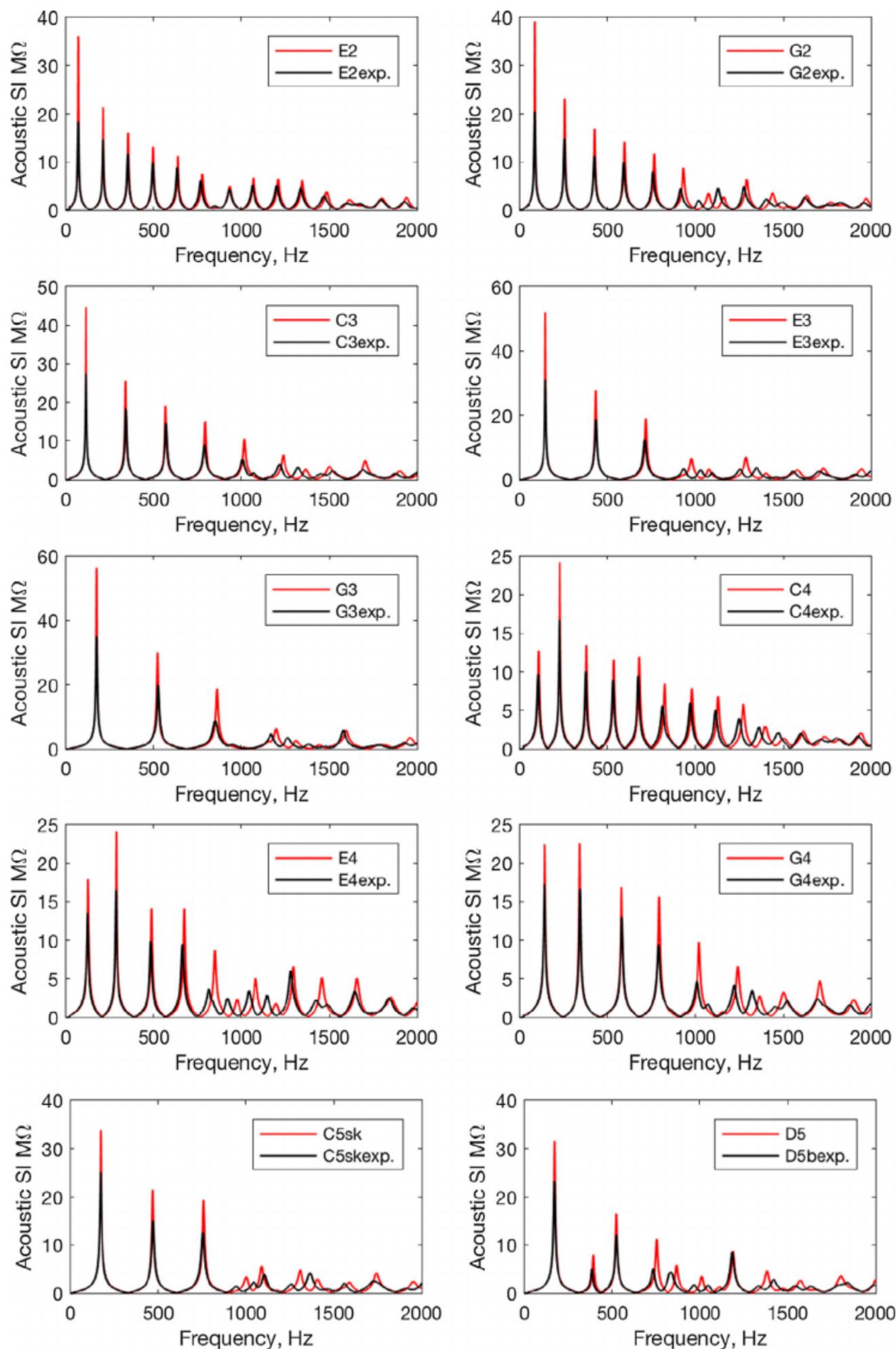
Fig. 2. Ten comparisons of experimental and computed results, in a (written) C major arpeggio from low written E2 up to C5 plus D5. Measured data are shown in black lines, calculated impedances in red lines. The measured and calculated lines largely overlap for each note, but the measured amplitudes are significantly lower and the frequencies very slightly lower. Note that for C4 to C5 the second impedance peak becomes the basis of the sound, through use of the speaker key, which depresses and shifts the first resonance out of a harmonic relationship with subsequent resonances. For D5, the sound becomes based upon the third resonance peak. The cutoff frequency is ~1000 Hz in this instrument; frequencies above this value are not expected to participate in the standing wave formation and in the feedback to the reed. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

difference can be ascribed to interactions between tone holes, which are expected to lower the resonance frequencies by a few cents [71]. Meanwhile, Fig. 4b shows the difference between the measured impedance peaks and the playing frequencies. These average at 37 ± 8 cents and correspond to the effects of the reed impedance.

Since the impedance peak differences between calculation and experiment are small and reasonably consistent, they appear to be systematic and might be reduced by further development of the computation, for example to take account of losses other than the viscous and heat losses inherent from a smooth-walled tube.
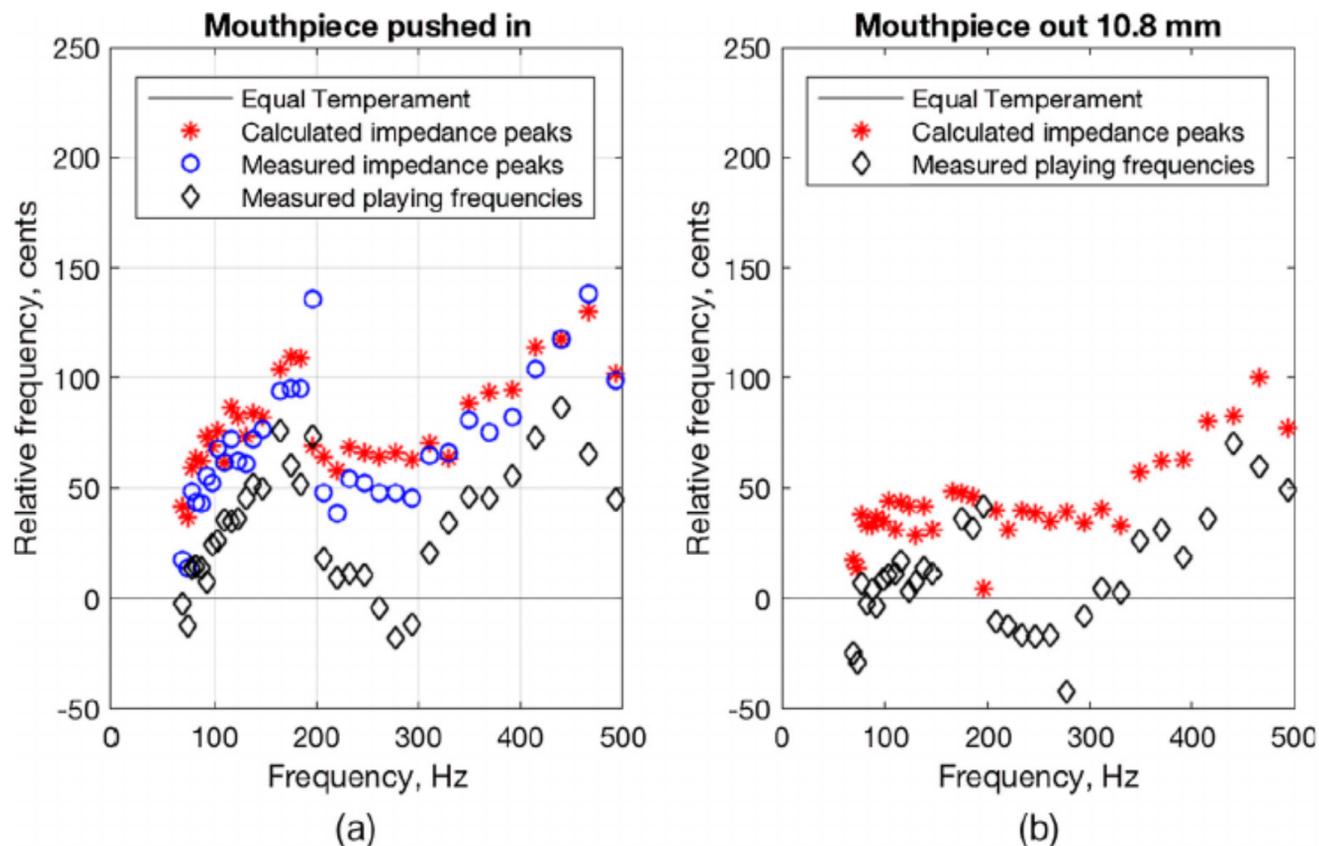
Fig. 3. Deviation in cents for each note, left: mouthpiece pushed in, right: mouthpiece pulled out 10.8 mm. The horizontal line at y = 0 represents equal temperament at A4 = 440 Hz. We did not make measurements of the "pulled-out" impedances. The 'break' in the instrument ranges between written Bb3 and B3 occurs at about 200 Hz and that between C5 and C#5 at about 450 Hz. Up to the first break the first resonance frequency is plotted, between the first and second break the second resonance and above the third break, the third resonance peak. Each data point corresponds to a single note. The equal temperament frequencies are calculated at A4 = 400 Hz.

Viscothermal losses due to porosity are the obvious candidates and indeed the Heckel instrument has a dry appearance and may need oiling. We note that much information on sound absorption of porous materials is available from the extensive literature on acoustic damping in architecture. However, an agreement within about 10 cents, which may be corrected empirically by an equivalent length of 3 mm on the mouthpiece segment, is sufficiently accurate for the research into historical instruments.

The difference of approximately 37 cents between the measured (or corrected calculated) peaks and the playing frequencies is ascribed to the embouchure correction discussed above. The results appear similar to those of Dalmont et al. [59] though there is more scatter, possibly because the latter used a blowing machine not a player. From the scaling expected, an embouchure equivalent length of about 15 ± 4 mm should be added to the top of the mouthpiece impedance. We therefore recalculated the impedances with a number of embouchure equivalent lengths added to the top of the column, just before the terminating impedance. There was substantial scatter but our estimate is that the equivalent length added onto the mouthpiece segment (at its same diameter) should be 17 ± 4 mm, plus 3 mm to correct the ~10 cent difference between our computed and measured impedance curves. The graph for 20 mm total added length is shown in Fig. 5 for both "mouthpiece pushed in" and "mouthpiece pulled out 10.8 mm". This shows that the embouchure and calibration correction works equally well for these two conditions. The assumption of frequency independence seems reasonable within our limited accuracy; there is some downwards trend in each register (which changes at about 200 and 450 Hz) but there is little overall frequency dependence. The value of + 17 mm for the embouchure correction is consistent with the soprano clarinet values of Dalmont et al., using our approximate scaling argument. They would vary somewhat with a different player and mouthpiece/reed, but the usefulness of this number is that, where the mouthpiece of an historical instrument is missing, we can make estimates of its effect based on the mouthpiece used in this investigation.

Note that a maker would not necessarily build the instrument so that the average deviation from playing pitch was zero, since notes that are flat are much harder for the player to correct than those that are sharp. Also, the player needs to be able to play in tune when the instrument is cold, especially for a doubling instrument such as a bass clarinet in A. A better choice is an instrument that is slightly sharp on average with no notes that are too flat to be easily corrected. The graphs above show that this is indeed the case for the Heckel instrument. Moreover, the consistent tendency for the intonation errors to rise fairly smoothly from the bottom to the top of each register makes it easier for the player to learn what adjustment is needed on each note.

5.2. Investigation of alternative fingerings

An important application of modelling in the understanding of historical instruments is comparative. For example, it is often of interest to study the intonation and stability of alternative fingerings (e.g. Cronin's work on bassoons [55]). We tested this by calculating and playing several notes that may have alternative fingerings: written Bb2, E b3, F3, C#4 and C5. These are referred to as 'normal' or 'fork' and are shown in Table 1. Only the calculated results are shown, again using a 17 mm embouchure correction and a 3 mm calibration correction.

The calculated impedance spectra for the notes are shown in Figs. 6–10. In each case the "normal" fingering is shown in black and the alternative in red. In all except Fig. 10, the first resonance and at least one other resonance aligns well between the two fingerings, and these also align with the fundamental and third harmonic of the intended played note (the harmonic positions are not shown on the figures). For some notes, the resonances align well with the 5th and 7th harmonics also. The observation on playing was that a two-resonance match was sufficient to produce a good match in intonation for the two fingerings, but that the more resonances that were aligned, the better was the match in timbre between the alternative fingerings.
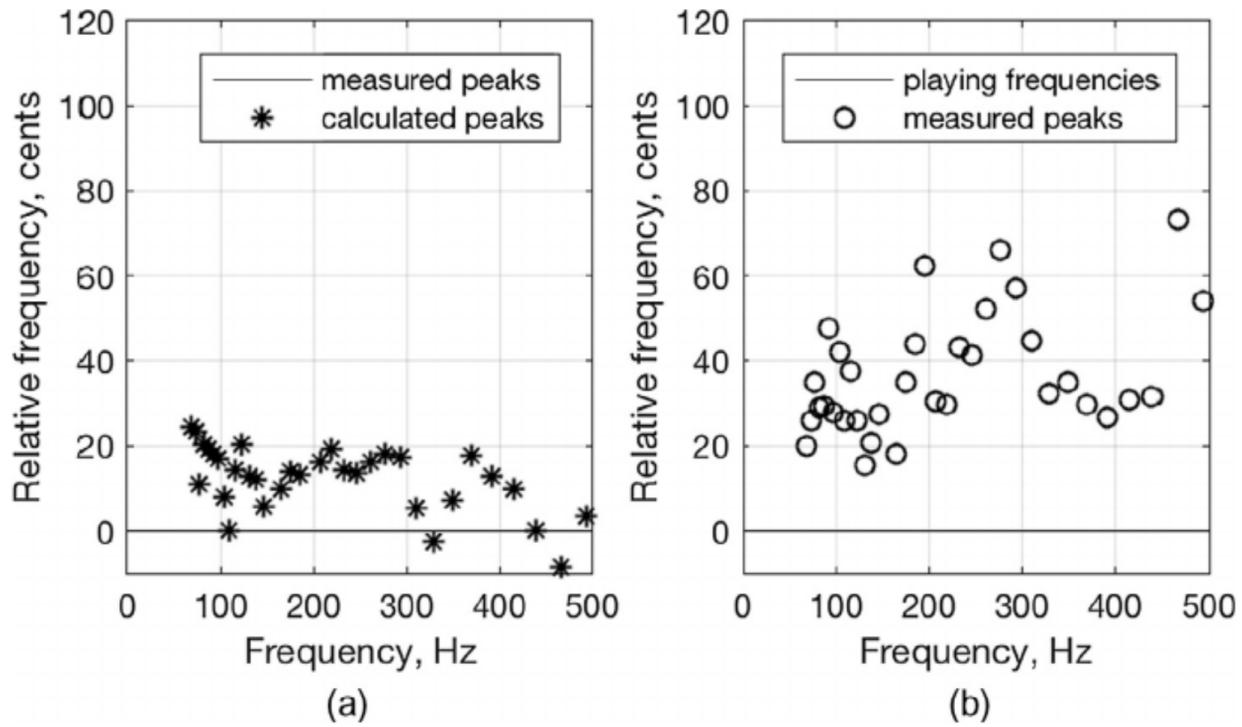
Fig. 4. (a): differences between calculated and measured impedance peaks. (b): differences between measured impedance peaks and measured playing frequencies. In both cases the mouthpiece was fully pushed in, and in case (b) was played at *mf* levels. No embouchure or calibration correction was applied to the calculated results in (a). The *y* = 0 line corresponds to zero difference.
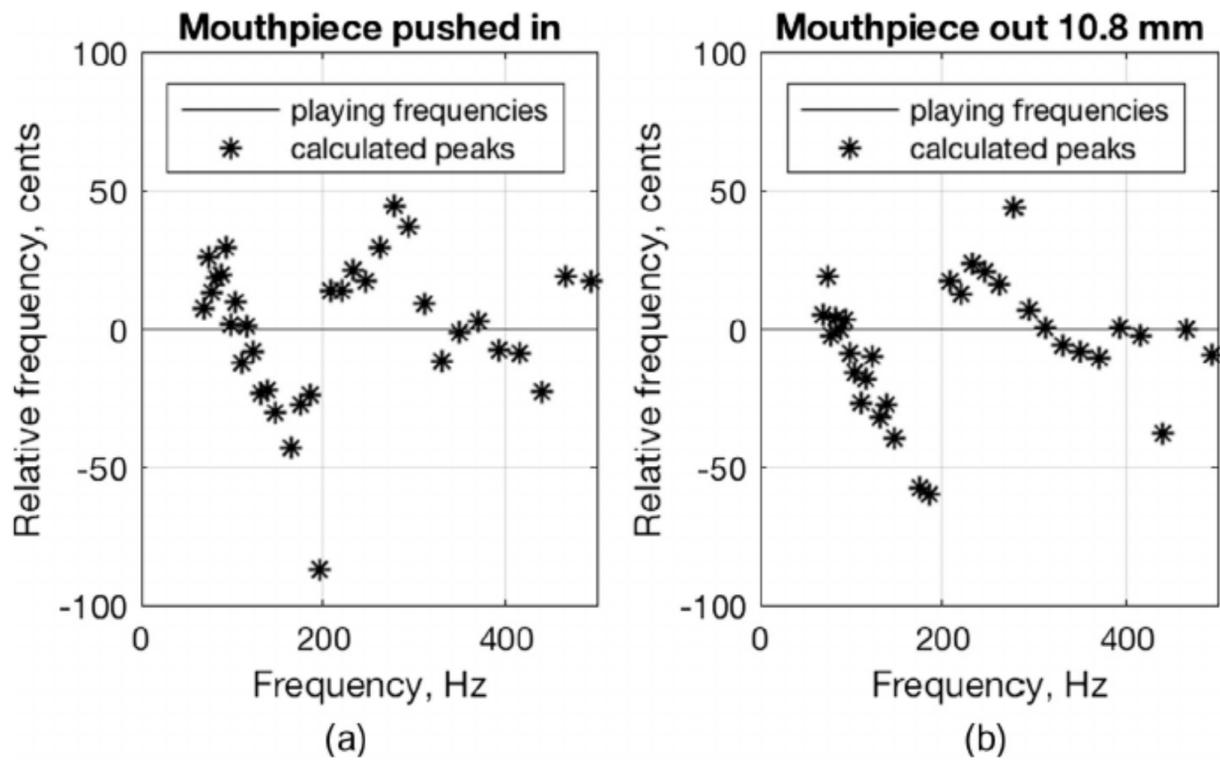

Fig. 5. Comparison between calculated impedance peaks and measured playing frequencies when the overall embouchure end correction was 20 mm. (a) with mouthpiece pushed in, (b) with mouthpiece pulled out.

Table 1
Alternative fingerings investigated. The fingering diagrams were constructed using the Brett Pimenthal Fingering Builder [84].

Note	Normal	Fork
Bb2		
Eb3		
F3		
C#4		
C5		
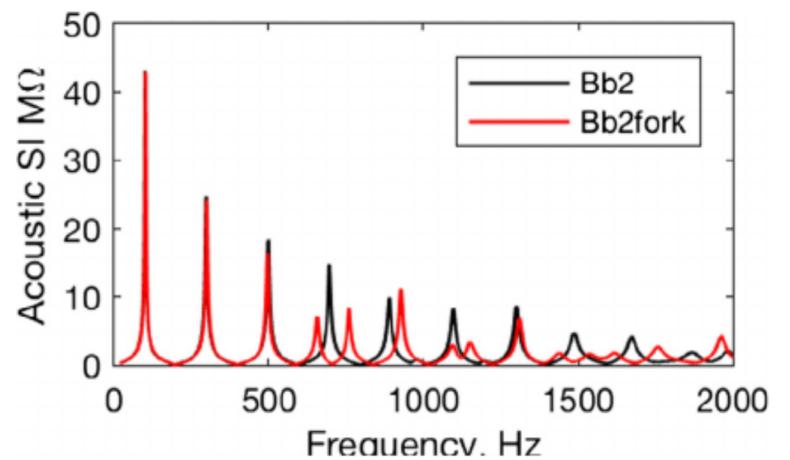
Fig. 6. . Calculated impedance spectra for two fingerings for the note Bb2. The first three resonances overlap almost exactly for the two fingerings.
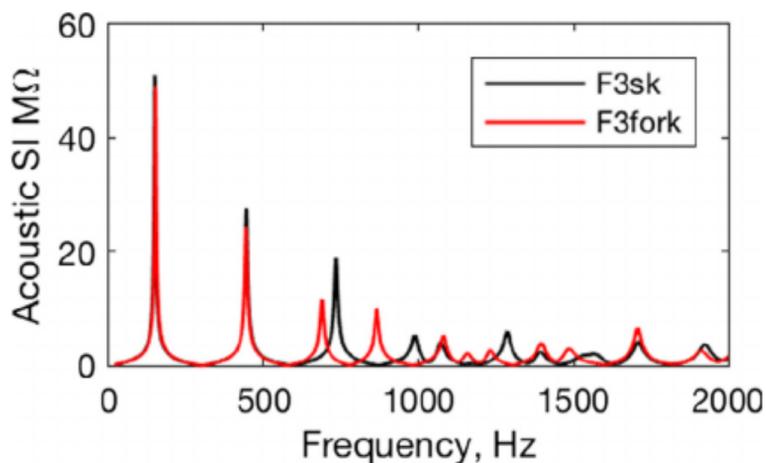
Fig. 7. Calculated impedance spectra for two fingerings for the note F3. The first two resonances overlap almost exactly.
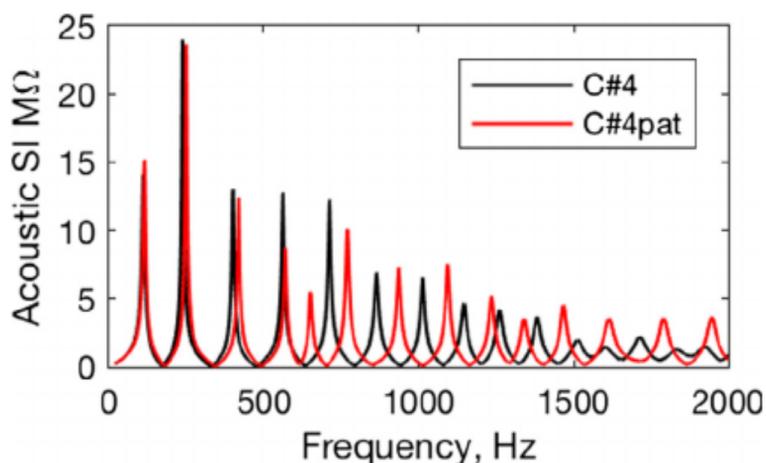
Fig. 8. Calculated impedance spectra for two fingerings for the note C#4. The "patent C#" fingering will be slightly sharp. Note that in the second register, the speaker key lowers and shifts the first resonance to a higher frequency, so the note produced becomes based on the second resonance frequency.
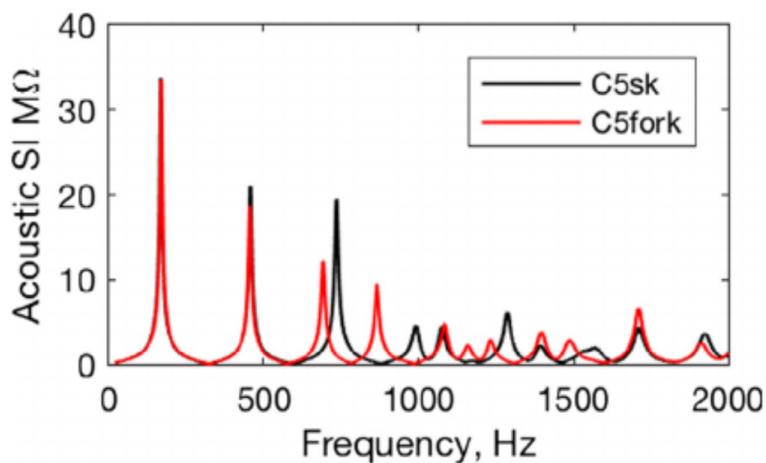

Fig. 9. Calculated impedance spectra for two fingerings for the note C5. The first two resonances overlap almost exactly. Again, it is the second resonance that aligns with the fundamental of the note played.

However, the forked D#/Eb3 shown in Fig. 10 showed a poor match between the two fingerings, and as predicted from the impedance curves, the fork fingering played almost a semitone sharp. Whilst the fork fingering is often acceptable for this note on earlier German system clarinets (sometimes it is the only fingering for this note) it is clearly not the case here (and is generally not the case for Albert system clarinets).

5.3. Impedance maps and the cutoff frequency

Jeltsch et al. [50] introduced the concept of impedance maps that show the resonances of all of the fingerings of the clarinet on one diagram and went on to apply it to experimental impedance

measurements. They do not give the method of calculation, but we have developed a similar procedure and applied it to both experimental and calculated impedances. The latter is shown in Fig. 11 for 3 mm embouchure correction, which should enable direct comparison with experiment.

The method of plotting is as follows. After calculating the impedance spectra for all fingerings of the instrument, the impedance data (as shown in e.g. Fig. 2) are analysed to determine all the peaks (resonances) in the spectra. It was sufficient to find the first seven peaks only in each impedance spectrum. For each note fingering and its corresponding set of resonances, the resonant frequencies are plotted with coordinates determined as:

- Ordinates: the nominal equal temperament frequency of the note, using (in this case) A4 = 440 Hz.
- Abscissae: the ratio of the actual frequency of the resonance to the nominal equal temperament frequency.
- The scales are logarithmic on both axes.
- Markers are placed at odd multiples of 1 on the horizontal axis.
- The cutoff frequency, 1000 Hz in this case, is also plotted in the same way (thus at an ordinate of 500 Hz its abscissa will be 2, while at an ordinate of 250 Hz its abscissa will be 4). It will be a straight line in this plot.
- In our case, peaks up to 2000 Hz are included in the plot.

The meaning of this map is that, for an equal-temperament clarinet, perfectly tuned at A4 = 440 Hz, we should see the points representing all the cooperating resonances for a given fingering lying along a set of vertical lines near the odd integers on the horizontal scale, up to the cutoff frequency. They should be displaced slightly to the right because of the necessary embouchure correction due to reed impedance, discussed earlier, but they should compare closely with the experimental measured impedances. The latter are shown in Fig. 12. The experimental map is constructed similarly, except that because of a small amount of noise in the experimental data, giving spurious peaks, each impedance spectrum is first processed to order the peaks by their 'prominence' [2] and then the first seven most prominent peaks are selected. This largely eliminates the spurious peaks. The register shifts on this instrument occur at approximately 200 and 450 Hz, and the change of resonance peak on which the note pitch is based is clearly seen.

To predict the actual playing pitches, the additional embouchure correction of 17 mm to allow for the reed impedance must be applied, as discussed in Section 5.1. The result is shown in Fig. 13. The resonances are now seen to be (mostly) very well aligned with the harmonic numbers. Thus, when the non-linear reed generator is combined with these impedance characteristics, a cooperative regime of oscillation will be set up [73], with each harmonically-related resonance frequency contributing to the stabilisation of the oscillation, up to the cutoff frequency. It can be seen that the first seven resonance frequencies are involved at the bottom of the instrument's range, but only one resonance frequency (the second or third) contributes near the top of the range. A slight sharpness is indicated as was actually found in the playing tests.

If we follow any particular resonance vertically, we see discontinuities occurring at 200 and 450 Hz, the register change points. At each discontinuity, a higher resonance takes over the role of determining the playing pitch; for example the first resonance is replaced by the second at abscissa 1 at ordinate 200 (written B3). It can also be seen that, as one moves up through the registers of the instrument, for each note fingering there are still resonances

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[2] The amount that the peak stands out due to its intrinsic height and its location relative to other peaks as defined in the MatLab ™ function findpeaks(array).
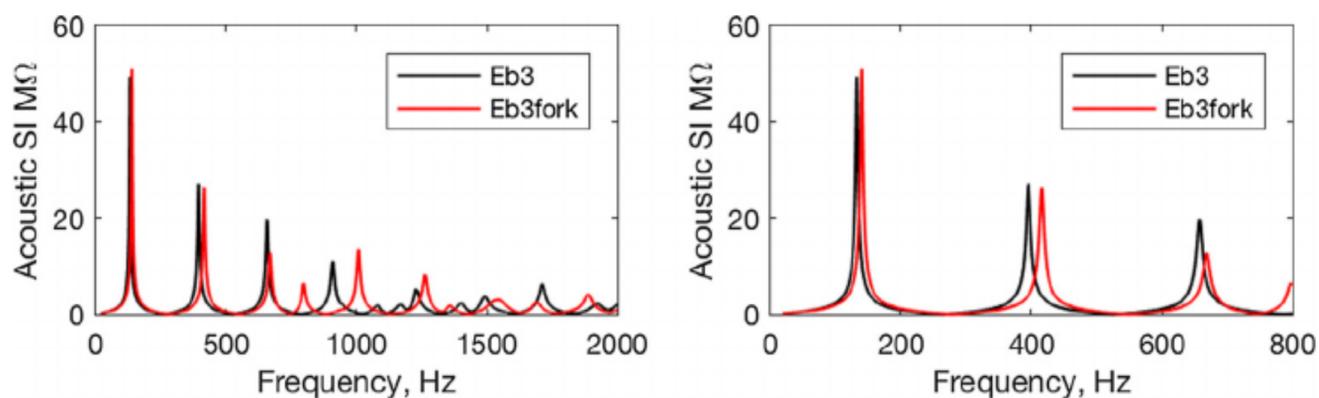
Fig. 10. Calculated impedance spectra for two fingerings for the note Eb3, at two frequency scales. The resonances indicate that the "alternative" fingering will be almost a semitone sharp.
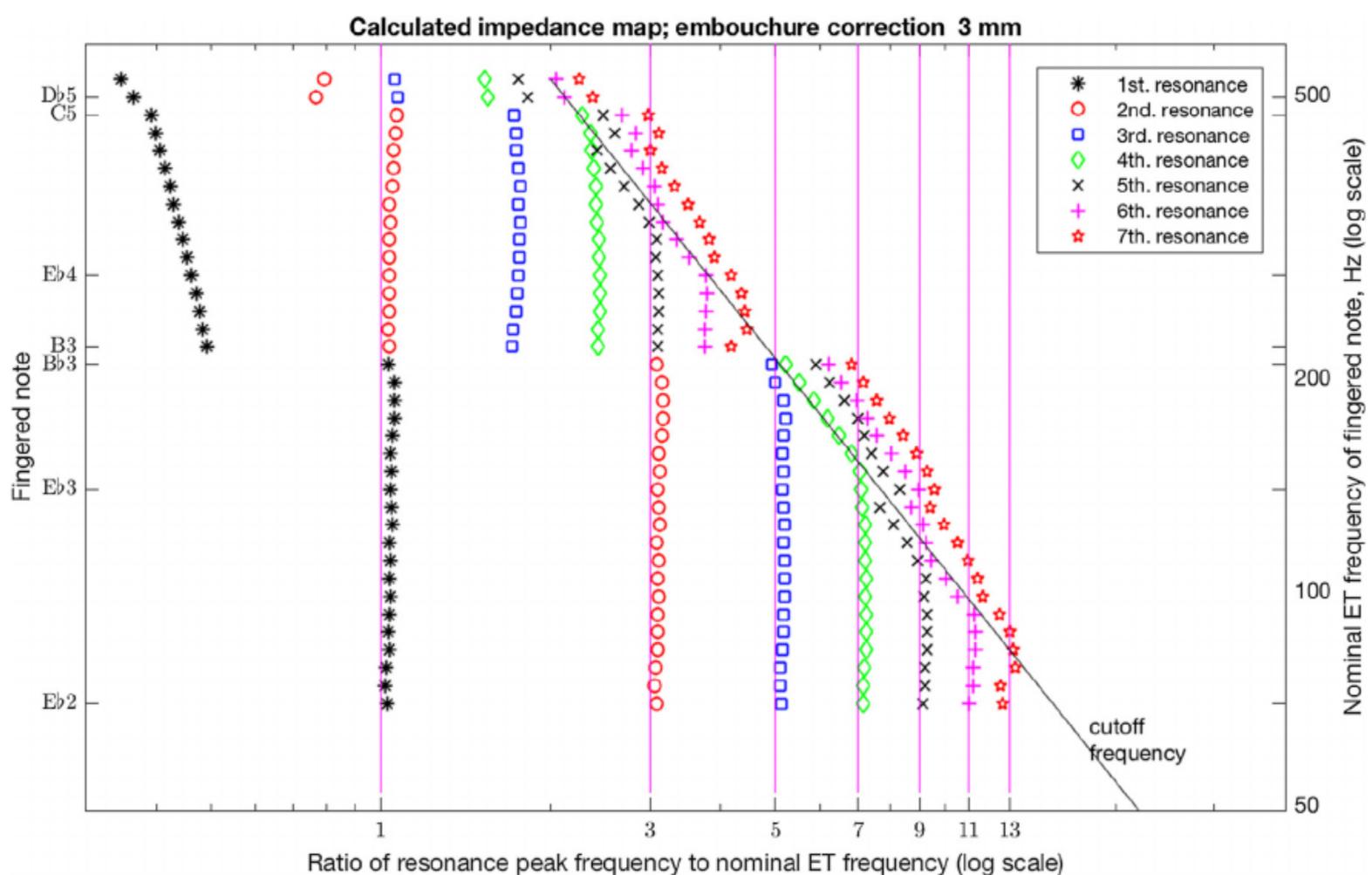

Fig. 11. Impedance map of calculated impedances, using an embouchure correction of 3 mm, which should enable good comparison with measured impedances. See main text for explanation of method of plotting.

which fall in the 1:3:5 etc. harmonic ratio, and thereby support harmonics of the played note.

Impedance maps also give a new insight into the nature of the cutoff frequency itself. If we look, for example, at the fourth resonance peak (denoted by green diamonds), we see that it runs vertically up to the point at which it intersects the line drawn at 1000 Hz, the approximate cutoff frequency from Benade's formula (Eq. (2)). Then the line turns sharply left to run parallel to the line tracing the cutoff frequency. Hence, although the resonance still exists, it ceases to be in a harmonic relationship with the first resonance peak for subsequent notes. At 200 Hz, the register key is applied and we see the discontinuity where the fourth peak moves to a lower frequency. This is now below cutoff, and the peak moves again along a vertical line. This does not correspond to any harmonic of the played note. However, if we instead follow the fifth resonance peak (denoted by black crosses), it can be seen that it initially supports the ninth harmonic of the played note. It then hits the cutoff and for the next few note pitches does not participate in the regime of oscillation. However, it then starts to support

the third harmonic at 200 Hz (B3). This view of the cutoff behaviour explains in a systematic way why higher notes may combine apparently random mixtures of resonance peaks in their regimes of oscillation. If we look at the line representing the third harmonic in the impedance maps, we see that as one moves through the notes towards the top of the instrument's range, the fifth, sixth and seventh resonances all successively play a role in supporting the third harmonic.

The locus of the resonance peaks, as they start to deviate from a harmonic relationship with the lower resonances, follows closely the slope of the cutoff frequency line. We can therefore identify, if not a precise cutoff frequency, then certainly a cutoff band. The impedance map (corrected for embouchure impedance) is thus seen to contain a great deal of information about the instrument: its tuning, its harmonicity, its likely mixture of partials and the degree with which they are aligned with the resonances of the instrument. We can in essence regard the impedance map as a "fingerprint" that characterises the acoustics of the instrument.
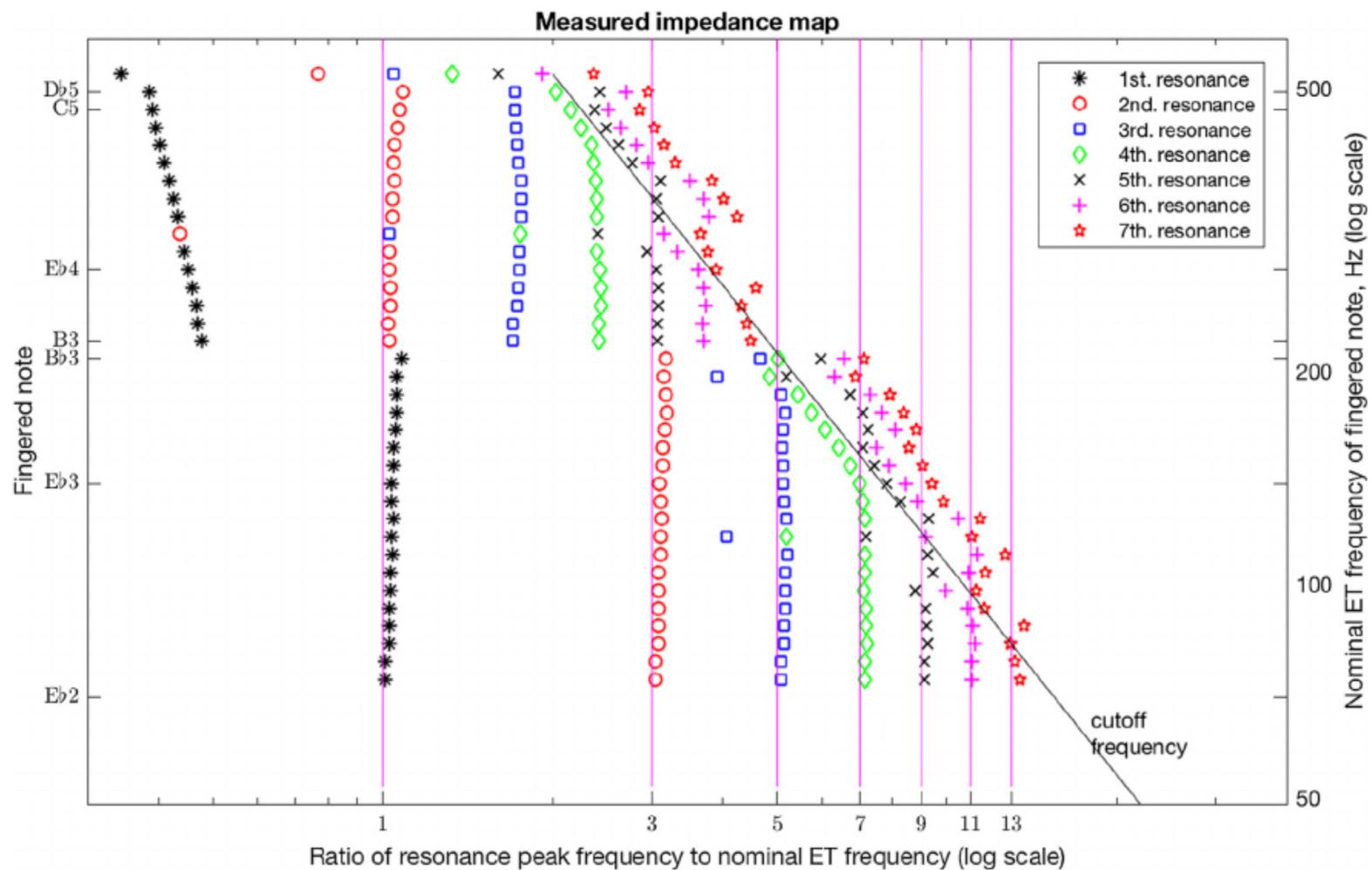
Fig. 12. Impedance map of experimental impedances, to compare with Fig. 11. Impedance map of calculated impedances, using an embouchure correction of 3 mm, which should enable good comparison with measured impedances. See main text for explanation of method of plotting.
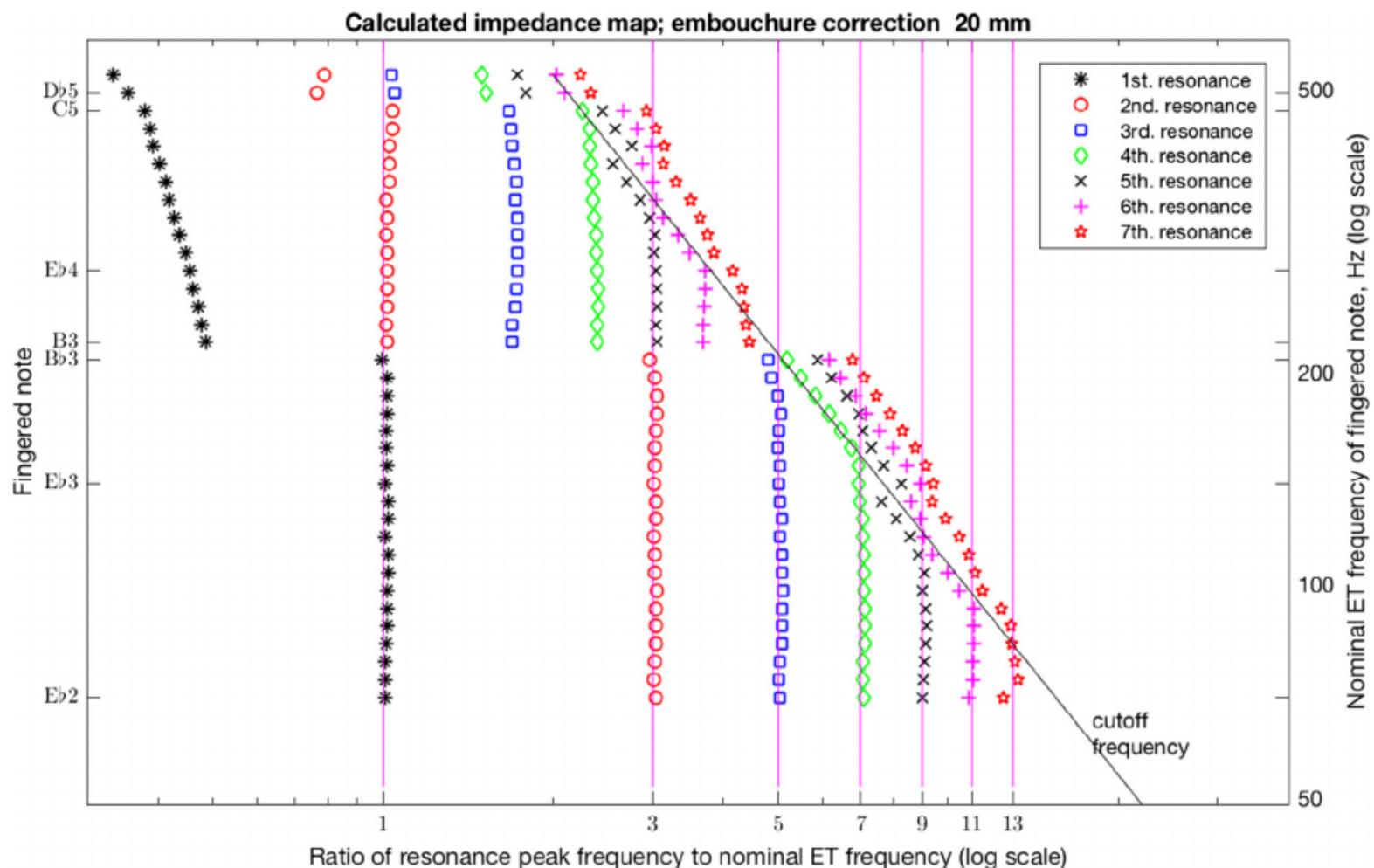

Fig. 13. Impedance map of calculated impedances, using an embouchure correction of 20 mm. This should indicate the actual audio frequencies on playing.

The cutoff band is seen more clearly in a simple linear plot of the individual resonance frequencies against the nominal ET frequencies of their fingerings, which we call a cutoff diagram, Fig. 14. There is no single cutoff for the whole instrument, but each resonance individually cuts off somewhere in the band 920–1320 Hz. Many of them cluster around 1000 Hz, indicating that the Benade approximation (Eq. (2)) is reasonable and useful, though not exact. Each resonance frequency increases continuously and then saturates, except at the register changes. At register changes, all resonances drop abruptly in frequency due to the
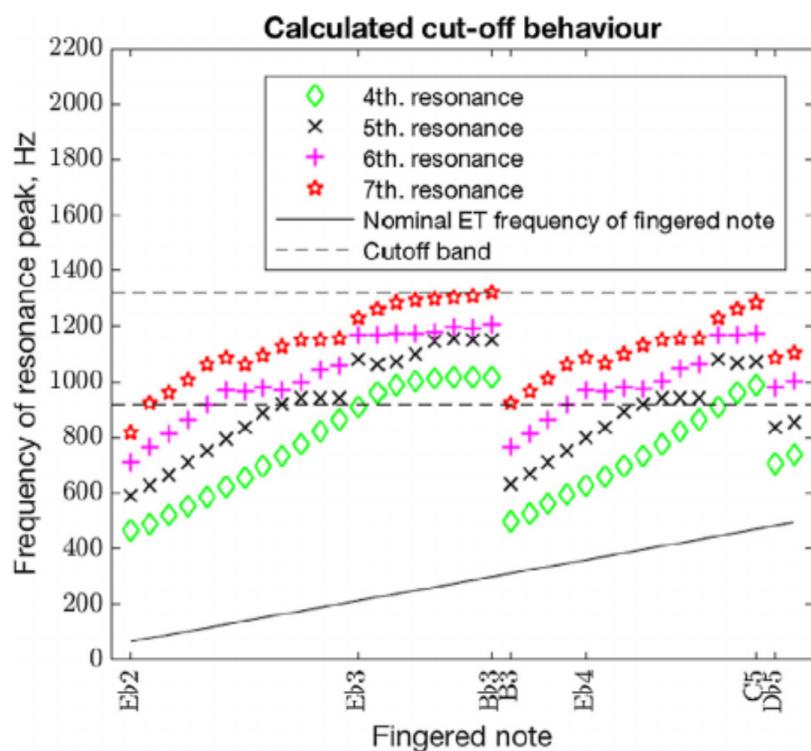
Fig. 14. Plot of 4th–7th resonances (supporting 7th–13th harmonics of the played note) against the nominal ET frequency of the fingered note. Note that frequencies in this diagram are absolute, not relative as in the abscissae of the impedance spectra. The first three resonances are below cutoff on this scale. Horizontal lines are drawn at 920 and 1320 Hz, representing the cutoff band. The discontinuities at x = 200 and 450 correspond to the register changes (Bb3 to B3 and C5 to C#5). Immediately after the register changes, all the resonances drop in frequency. Calculated with total embouchure correction of 20 mm, corresponding to the impedance map of Fig. 13. We term this plot a 'cutoff diagram'.

effect of the speaker key and (for the third register) first fingerhole opening. The cutoff is of course not abrupt, but a roll-off, and it is seen to depend on the fingering. The onset of cutoff for each resonance will be at a different fingering. It will depend on the spacing, diameters and chimney lengths of the holes. This is consistent with the observations of Moers and Kergomard [85].

We have not displayed the information contained in the peak heights, since the plot becomes complicated and adds little insight. We simply note that the peak heights near cutoff become quite small, as can be seen in Fig. 2, and also that the height of the first resonance peak (abscissa 1 on the impedance maps) drops by about 50% on changing from the first to the second register ('crossing the break').

6. Conclusions and future work

The computational model used in this study is based on small-signal, linear, plane- and spherical-wave acoustics with viscous and thermal losses at smooth walls. It does not take account of some loss mechanisms such as wall porosity, internal tone-hole edge turbulence and finger and pad resilience. Nevertheless, it is remarkably accurate for predicting the absolute values of resonance frequencies and the relative heights of resonance peaks. We conclude that the method is certainly accurate enough for the purpose of reconstructing the acoustic impedance (resonance) spectra of instruments of this type. This extends the similar conclusion of Dalmont et al. [59] from soprano clarinets, oboes and alto saxophones to bass clarinets, and provides a reasonably accurate measurement of the embouchure equivalent length in the instrument studied.

It may eventually be possible to reconstruct the entire sound, using methods pioneered by Taillard and his associates [86], which requires also the more detailed non-linear treatment of the reed/mouthpiece generator, but the preliminary step for that process is the measurement or calculation of input impedances to sufficient accuracy. We believe that we achieve tuning accuracy (after

corrections applied) at worst within a few cents, which is entirely adequate to measure the pitch and temperament at which an instrument was designed to play. The relative accuracy within or between instruments would be much better, so there is no problem in comparing alternative fingerings for notes, for determining the pitch and temperament in which the instrument was constructed or for comparing the overall acoustic behaviour of two different instruments. In this project, one of the authors (DKB) has a particular interest in comparing bassoon-form and straight-form bass clarinets of the early nineteenth century, and the method appears to be very suitable.

As pointed out by many others [10,30,34,53,55,57,59] the prediction of resonance peaks has utility in instrument design, restoration and modification. The effect of drilling or moving a hole, or of reaming the bore (for example, for removing the tenon compression induced by tenon lapping before cork came into use [87]) can be checked before material is removed. The Heckel under study in fact has a centre tenon that has a 0.1 mm restriction for a few centimetres, but calculation of its effect on the impedances showed that it had a quite negligible influence and so did not need any reaming. Playing problems with a particular instrument may also be diagnosed. Again, it is clear that the Heckel instrument would play more in tune with a slightly longer neck, or at a higher orchestra pitch. Examination of the neck does indicate that it might be a later replacement and not an original. The calculated impedances could also indicate how to alter a tone hole to improve the tuning, and what effect this would have on other notes. We believe, therefore, that we have quantitatively validated the computational method of acoustic impedance modelling as a research tool for investigating and restoring both modern and historical bass clarinets.

References

[1] http://minim.ac.uk; consulted November 13 2017
[2] ICOM Code of Ethics, article 1. http://bit.ly/2k4rm84. Consulted 8 March 2018.
[3] Karp Cary. Restoration, conservation, repair and maintenance: some considerations on the care of musical instruments. Early Music 1979;79–84 .
[4] Bowen, Keith. 'Vergessene Klänge und kommende Sounds. »Wagners Bassklarinette: Klang und Zeit« – Symposium an der Robert Schumann Hochschule in Düsseldorf 15. Juni 2013. rohrblatt 4 181-2; 2013.
[5] ICOM/CIMCIM: Recommendations for regulating the access to musical instruments in public collections: 1985. http://bit.ly/2ADcQfz.
[6] Rice Albert R. The Baroque Clarinet. Oxford: Clarendon Press; 1992.
[7] Rice. The Clarinet in the Classical Period. Oxford: Oxford University Press; 2003.
[8] Rice. From the Clarinet D'amour to the Contra Bass: A History of the Large Size Clarinets, 1740-1860. OUP USA; 2009.
[9] Campbell, Murray and Clive Greated, The musician's guide to acoustics. New York: Schirmer, p259 et seq; 1987.
[10] Benade AH. Fundamentals of Musical Acoustics New York: Dover. Corrected reprint of 1976 edition. New York: Oxford University Press; 1990 .
[11] Boutin, Henri, Sandie Le Conte, Benoit Fabre, Jean-Loïc Le Carrou. 'Influence of the Surface Condition in Bore of Woodwind Instruments on the Acoustic Impedance'. Wood Musick Conference, London: RCM; 2015.

http://woodmusick.org/wp-content/uploads/COST%20Conference%20booklet%20London.pdf.

[12] Smith RA, Mercer DMA. Possible causes of woodwind tone colour. J Sound Vib 1974;32:347–58.

[13] Benade Arthur H. Horns, strings and harmony. New York: Anchor; 1960.

[14] Carral Sandra, Vergez Christophe, Nederveen Cornelis J. Toward a single reed mouthpiece for an oboe. Archiv Acoust 2011;36:267–82 .

[15] Helmholtz, Hermann von. Tonempfindungen. (1863). Tr. Alexander J. Ellis. On the sensations of tone. (1954) New York: Dover reprint.

[16] Bouasse, H. Instruments à Vent (Vol. I and II). Paris: Librairie Delagrove; 1929.

[17] Lamb H. Dynamical theory of sound. London: E. Arnold; 1910.

[18] Olson HF. Acoustical engineering. Princeton: Van Nostrand; 1957.

[19] Caussé R, Kergomard J, Lurton X. Input impedances of brass musical instruments. J Acoust Soc Am 1984;75:241–54 .

[20] Backus J. Small vibration theory of the clarinet. J Acoust Soc Am 1963;35:305–13.

[21] Strutt JW. The theory of sound (2 nd Baron Rayleigh). London: Macmillan and Co; 1877.

[22] McGinnis CS, Gallagher C. The mode of vibration of a clarinet reed. J Acoust Soc Am 1941;12:529–31.

[23] Taillard Pierre-André, Kegomard Jean. An analytical prediction of the bifurcation scheme of a clarinet-like instrument: effects of resonator losses. Acta Acustica united with Acustica 2015;101:279–91 .

[24] Levine Harold, Schwinger Julian. On the radiation of sound from an unflanged circular pipe. Phys Rev 1948;73:383–406 .

[25] Dalmont J-P, Nederveen CJ, Joly N. Radiation impedance of tubes with different flanges: numerical and experimental investigations. J Sound Vib 2001;244 (3):505–34.

[26] Worman, Walter E. Self-Sustained Nonlinear Oscillations in Clarinet-Like Systems. Ph.D. diss., Case Western Reserve University, Cleveland, Ohio; 1971.

[27] Chaigne Antoine, Kergomard Jean. Acoustics of musical instruments (1 st English edition). New York: Springer-Verlag; 2016.

[28] Plitnik GR, Strong WJ. Numerical method for calculating input impedances of the oboe. J Acoust Soc Am 1979;65:816–25 .

[29] Keefe DH. Woodwind air column models. J Acoust Soc Am 1990;88:35–51.

[30] Nederveen CJ. Acoustical aspects of woodwind instruments (revised edition). Dekalb, IL: Northern University Illinois Press; 1998 .

[31] Guillemain P, Kergomard J, Voinier T. Real-time synthesis of clarinet-like instruments using digital impedance models. J Acoust Soc Am 2005;118:483–94.

[32] Scavone Gary P, Smith Julius O. A stable acoustic impedance model of the clarinet using digital waveguides Sep. 18-20. Proc. Of the 9 th International Conference on Digital Audio Effects (DAFx-06), Montreal, Canada, 2006 .

[33] Bolton, Philippe. 'Resonans: a software program for developing new wind instruments. FOMRHI QUARTERLY, 79 69-72; 1995 (Communication No. 1356).

[34] Scavone Gary Paul. An acoustic analysis of single reed woodwind instruments with an emphasis on design and performance issues and digital waveguide modeling techniques PhD Thesis. Stanford University; 1997 .

[35] Yong Shi. Comparing Theory and Measurements of Woodwind-Like Instrument Acoustic Radiation MA Thesis. Montreal: McGill University; 2009 .

[36] Benade AH, Kouzoupis SN. The clarinet spectrum: theory and experiment. J Acoust Soc Am 1988;83:292–304 .

[37] Benade Arthur H, Keefe Douglas H. Galpin Soc J 1996;49:113–42.

[38] Benade, A.H. 'Benade's NX clarinet: Its genesis' (written in 1987 and published posthumously), The Clarinet; Jan/Feb 1994, p. 46–8.

[39] Jameson, George. 'Benade's clarinet: Mechanical and other considerations. The Clarinet; May/June 1994, p. 32–3.

[40] Stephen Fox Clarinets. http://www.sfoxclarinets.com/Benade.html. Consulted 8 March 2018.

[41] Dalmont J-P. Acoustic impedance measurement, part i: review. J Sound Vib 2001;243:427–39.

[42] Dalmont J-P. Acoustic impedance measurement, part ii: review. J Sound Vib 2001;243:441–59.

[43] Kausel, W. and Kuehnelt H. 'A practical way to measure intonation quality of woodwind instruments using standard equipment without custom made adaptors'. Acoustics '08, Paris. (www.acoustics08-paris.org).

[44] Widholm G., Pichler H., Ossmann T. 'BIAS: a computer-aided test system for brass wind instruments'. An Audio Engineering Society Preprint; 1989, 2834.

[45] https://www.ircam.fr/project/detail/pafi/consulted 22 November 2017.

[46] Sharp DB. Acoustic pulse reflectometry for the measurement of musical wind instruments PhD Thesis. The University of Edinburgh; 1996 .

[47] Sharp DB. Increasing the length of tubular object that can be measured using acoustic pulse reflectometry. Meas Sci Technol 1998;9:1469–79 .

[48] Li A, Sharp DB, Forbes BJ. Increasing the axial resolution of bore profile measurements made using acoustic pulse reflectometry. Measur Sci Technol 2005;16:2011–9.

[49] Campbell, Murray "Acoustical Evaluation of Historic Wind Instruments" Forum Acusticum 2005 Budapest; Acta acustica united with Acustica, 91 Supplement; 2005.

[50] Jeltsch Jean, Gibiat Vincent, Forest L. Acoustical Study of a Set of Six Key Baumann's Clarinets. In: Proc. International Symposium on Musical Instruments (Dourdan, 1995). p. 134–40 .

[51] Lefevre Jean-Xavier, de Clarinette Méthode. Adopté Par Le Conservatoire Pur Servir À l'étude Dans Cet Établissement. Paris: Impr. du Conservatoire de Musique; 1802 .

[52] Jeltsch, Jean and Nicholas Shackleton, "Caractéristation Acoustique de Trois Clarinettes de Facteurs Lyonnais," in Colloque Acoustique et Instruments Anciens: Factures, Musiques et Science (Colloque Acoustique et Instruments Anciens: Factures, Musiques et Science, Paris: Cité de la Musique, 1999), p. 103–24.

[53] Schumacher RT. Ab Initio calculations of the oscillations of a clarinet. Acustica 1981;48:71–85.

[54] Backus J. Input impedance for reed woodwind instruments. J Acoust Soc Am 1974;56:1266–79.

[55] Cronin Robert H. Understanding the operation of auxiliary fingerings on the modern bassoon. J Int Double Reed Soc 1996;24:13–30 .

[56] Cronin, Robert and Douglas Keefe. 'Understanding the operation of auxiliary fingerings on conical doublereed instruments'. Unpublished notes for a talk presented at the 131st meeting of the Acoustical Society of America, Indianapolis, Indiana; 13-17 May 1996.

[57] Dart Mathew. The Baroque Bassoon: form, construction, acoustics, and playing qualities. London: London Metropolitan University; 2011. PhD thesis .

[58] Hichwa, Bryant and David Rachor. 'In-depth acoustic modeling and temperament studies of 18th and early 19th century baroque bassoons comparing originals and reproductions by maker, time period, and region'. Société Francaise d'Acoustique. Acoustics 2012, Apr 2012, Nantes, France; 2012. Online: <hal-00810841>

[59] Dalmont JP, Gazengel B, Gilbert J, Kergomard J. Some aspects of tuning and clean intonation in reed instruments. Appl Acoust 1995;46:19–60 .

[60] Mamou-Mani A, Sharp DB. Evaluating the suitability of acoustical measurement techniques and psychophysical testing for studying the consistency of musical wind instrument manufacturing. Appl Acoust 2010;71:668–74.

[61] Mamou-Mani A, Sharp DB, Meurisse T, Ring W. Investigating the consistency of woodwind instrument manufacturing by comparing five nominally identical oboes. J Acoust Soc Am 2012;131(1):728–36 .

[62] Kowal, Paulina; Sharp, David and Taherzadeh, Shahram. 'Analysing differences between the input impedances of five clarinets of different makes'. In: Institute of Acoustics Annual Spring Conference 2013: Acoustics 2013, 13 May 2013, Nottingham, UK; 2013.

[63] Hélie Thomas, Rodet Xavier. Radiation of a pulsating portion of a sphere: application to horn radiation. Acta Acustica United with Acustica 2003;89:565–77.

[64] Benade AH, Murday JS. Measured end corrections for woodwind tone holes. J Acoust Soc Am 1967;41:1609.

[65] Fletcher Neville H, Rossing Thomas D. The physics of musical instruments. 2nd ed. New York: Springer; 2010.

[66] Kulik Yakov. Transfer matrix of conical waveguides with any geometric parameters for increased precision in computer modeling. J Acoust Soc Am 2007;122(5):EL179.

[67] Grothe Thimo. Experimental Investigation of Bassoon Acoustics Dr,-Ing thesis. Technical University of Dresden; 2014 .

[68] Nederveen, C.J., J. K. M. Jansen, and R. R. van Hassel, "Corrections for Woodwind Tone-Hole Calculations", Acta Acustica United with Acustica 84, no. 5 (September 1, 1998), p. 957–66.

[69] Dubos V, Kergomard J, Khettabi A, Dalmont J-P, Keefe DH, Nederveen CJ. Theory of sound propagation in a duct with a branched tube using modal decomposition. Acta Acustica (Stuttgart) March 1999;85(2):153–69 .

[70] Dalmont J-P, Nederveen CJ, Dubos V, Ollivier S, Vincent Meserette V, te Sligte E. Experimental determination of the equivalent circuit of an open side hole: linear and non linear behaviour. Acta Acustica united with Acustica 2002;88:567–75. 9 .

[71] Lefebvre Antoine, Scavone Gary P, Kergomard Jean. External tonehole interactions in woodwind instruments. Acta Acustica United with Acustica 2013;99:975–85.

[72] Thompson Stephen C. The effect of the reed resonance on woodwind tone production. J Acoust Soc Am 1979;66:1299–307 .

[73] Benade AH, Gans DJ. Sound production in wind instruments. Ann N Y Acad Sci 1968;155:247–63.

[74] Reiter Edith. Wilhelm Heckel, six generations dedicated to music. Wiesbaden: Marixverlag; 2014 .

[75] Bowen, D. Keith. 'The Rise and Fall of the Bass Clarinet in A', The Clarinet; September 2011, p. 44-1; 2011.

[76] Félix Simon, Dalmont Jean-Pierre, Nederveen CJ. Effects of bending portions of the air column on the acoustical resonances of a wind instrument. J Acoust Soc Am 2012;131(5):4164–72. https://doi.org/10.1121/1.3699267.

[77] Bär, Frank P. Verzeichnis der Europäischen Musikinstrumente in Gemanischen Nationalmuseum Nürnberg. Band 6: Liebesklarinetten, Bassetthörner, Bassklarinetten, Metallklarinetten. Wilmhelmshaven: Florian Noetzel; 2006.

[78] Sharp D, Mamou-Mani A, Van Walstijn M. A single microphone capillary-based system for measuring the complex input impedance of musical wind instruments. Acta Acust United Ac 2011;97. 819–829(5) .

[79] Widholm G, Pichler H, Ossmann T. BIAS A computer-aided test system for brass wind instruments. Audio Eng Soc 1989 .

[80] Widholm G, Winkler W. Evaluation of musical instrument quality by computer systems ISBN: 91845289876. In: Examples of realisation', Proceedings of the SMAC93. Royal Swedish Academy of Music; 1994. p. 560–5 .

[81] Widholm G. Brass wind instrument quality measured and evaluated by a new computer system Trondheim, Norway (June 26–30, 1995). In: Proceedings of the 15th International Congress on Acoustics. p. 517–20 .

[82] Kausel W. Bore reconstruction of tubular ducts from its acoustic input impedance curve. IEEE Trans Instrum Meas August 2004;53(4):1097–105. https://doi.org/10.1109/TIM.2004.831440.

[83] de Cheveigne Alain, Kawahara Hideki. YIN a fundamental frequency estimator for speech and music. J Acoust Soc Am 2002;111:1917–30 .

[84] Pimentel, Brett, Fingering Diagram Builder, https://fingering. bretpimentel.com/#!/clarinet-german/albert/. Consulted 11 March 2018.

[85] Moers Elise, Kergomard Jean. On the cutoff frequency of clarinet-like instruments. Geometrical versus acoustical regularity. Acta Acustica United with Acustica 2011;97:984–96. https://doi.org/10.3813/AAA.918480.

[86] Taillard, Pierre-André, Fabrice Silva and Philippe Guillemain, 'Simulation en temps réel de l'impédance d'entrée mesurée ou calculée des instruments à vent'. Conference CFA/VISHNO April 2016; available on ResearchGate.

[87] McGee, Terry. 'Effect of thread wrapping on flute tenons' http://www.mcgee-flutes.com/effects_of_thread_wrapping.htm (2011). Consulted 20/11/2017.